Excited Fermion Contribution to Z^0 Physics at One Loop

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Abstract

We investigate the effects induced by excited leptons at the one-loop level in the observables measured on the Z peak at LEP. Using a general effective Lagrangian approach to describe the couplings of the excited leptons, we compute their contributions to both oblique parameters and Z partial widths. Our results show that the new effects are comparable to the present experimental sensitivity, but they do not lead to a significant improvement on the available constraints on the couplings and masses of these states.

I. INTRODUCTION

The standard model of electroweak interactions (SM) is not able to give a satisfactory explanation to family repetition and to the complex pattern of the fermion masses. One expects a substantial improvement in the understanding of these problems when considering an underlying fermionic substructure where the usual fermions share some constituents (preons) [1]. In this sense, the SM would be just the low-energy limit of a more fundamental theory, being valid only at energies below the compositeness mass scale Λ.

One of the most unambiguous predictions of the composite models is the existence of an excited lepton state for each known lepton. Unfortunately, we do not yet have a satisfactory model that could reproduce the whole family spectrum. In view of the lack of a unique predictive theory, a model-independent phenomenological analysis of the effects of fermion compositeness seems the most appealing approach. On this ground, we can employ the effective Lagrangian techniques to describe the physics of these excited states below the compositeness scale.

This approach has been employed in several phenomenological studies that analysed the expected signatures of these excited fermions in pp [2,3], e^+e^- [2,4,5,6,7], and ep [4,6] collisions at high energies. On the experimental side, several searches for these particles have been carried out, including those at the CERN Large Electron–Positron Collider (LEP) [8] and at HERA [9]. At LEP, the experiments at the Z pole excluded the existence of excited spin–$\frac{1}{2}$ fermions with mass up to 46 GeV from the pair production search ($e^+e^- \to \ell^*\ell^*$), and up to 90 GeV from direct single production ($e^+e^- \to \ell\ell^*$) for a scale of compositeness $\Lambda < 2.5$ TeV [8]. Very recent results from the L3 Collaboration [11], at centre-of-mass energies of 130–140 GeV, determined the lower mass limits at 95% C.L. of 64.7 GeV for the excited electrons, and roughly $\Lambda \geq 1.4$ TeV for $90 \leq M_{e^*} \leq 130$ GeV. The experiments at the DESY ep collider HERA also searched for resonances in the $e\gamma$, νW, and eZ systems [9,10]; however the LEP bounds on excited leptons couplings are about one order of magnitude more stringent in the mass region below the Z mass.

In spite of the failure of all the direct searches for compositeness, we could expect that the next generation of accelerators, working at higher centre-of-mass energies, would be able to obtain a direct evidence of the existence of these composite states. On the other hand, an

important source of indirect information about new particles and interactions is the precise measurement of the electroweak parameters done at LEP. Virtual effects of these new states can alter the SM predictions for some of these parameters and the comparison with the experimental data can impose bounds on their masses and couplings.

In this work we investigate the one-loop effects of excited leptons in the observables measured on the Z peak at LEP. Using an effective Lagrangian in terms of dimension five operators to describe the couplings of the excited leptons, we compute their contribution to both oblique and vertex corrections to the electroweak parameters. Our results show that the new effects are comparable to the present experimental sensitivity, but they are only able to constrain very marginally the model parameters beyond the present limits from direct searches.

The outline of the paper is as follows. In section II, we introduce the effective Lagrangian describing the couplings of the excited leptons. Section III contains the relevant analytical expressions for the one-loop corrections induced by the excited leptons. Our results and their respective discussion are given in Section IV. This paper is supplemented with Appendix A where we list all the relevant Passarino–Veltman functions.

II. EFFECTIVE INTERACTIONS

In order to reduce the number of free parameters in a general effective Lagrangian, we concentrate here in a specific model, following the formulation of Hagiwara *et al.* [4]. This particular model has been used by several experimental collaborations as a guideline to the search of composite states. We consider excited fermionic states with spin and isospin $\frac{1}{2}$, and we assume that the excited fermions acquire their masses before the $SU(2) \times U(1)$ breaking, so that both left-handed and right-handed states belong to weak isodoublets. We introduce the weak doublets, with hypercharge $Y = -1$, for the usual left-handed fermion (ψ_L) and for the excited fermions (Ψ^*),

$$\psi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad , \text{ and } \quad \Psi^* = \begin{pmatrix} N \\ E \end{pmatrix} ,$$

The most general dimension-five effective Lagrangian describing the coupling of the excited

fermions to the usual fermions, which is $SU(2) \times U(1)$ invariant and CP conserving can be written as [4]

$$\mathcal{L}_{Ff} = -\frac{1}{2\Lambda}\bar{\Psi}^*\sigma^{\mu\nu}\left(gf_2\frac{\tau^i}{2}W^i_{\mu\nu} + g'f_1\frac{Y}{2}B_{\mu\nu}\right)\psi_L + \text{h. c.}, \tag{1}$$

where f_2 and f_1 are weight factors associated to the $SU(2)$ and $U(1)$ coupling constants, with Λ being the compositeness scale, and $\sigma_{\mu\nu} = (i/2)[\gamma_\mu, \gamma_\nu]$. g and g' are the gauge coupling constants of $SU(2)$ and $U(1)$ respectively. At tree-level they can be expressed in terms of the electric charge, e, and the Weinberg angle, θ_W, as $g = e/\sin\theta_W$ and $g' = e/\cos\theta_W$. We will assume a pure left-handed structure for these couplings in order to comply with the strong bounds coming from the measurement of the anomalous magnetic moment of leptons [12].

In terms of the physical fields, the Lagrangian (1) becomes

$$\mathcal{L}_{Ff} = -\sum_{V=\gamma,Z,W} C_{VFf}\bar{F}\sigma^{\mu\nu}(1-\gamma_5)f\partial_\mu V_\nu - i\sum_{V=\gamma,Z} D_{VFf}\bar{F}\sigma^{\mu\nu}(1-\gamma_5)fW_\mu V_\nu + \text{h. c.}, \tag{2}$$

where $F = N, E$, and $f = \nu, e$. The non-abelian structure of (1) gives rise to a contact quartic interaction, such as the second term in the r.h.s. of Eq. (2). In this equation, we have omitted terms containing two W bosons, which do not play any role in our calculations. C_{VFf} is the coupling of the vector boson with the different kinds of fermions,

$$\begin{align}
C_{\gamma Ee} &= -\frac{e}{4\Lambda}(f_2 + f_1) & , \quad C_{\gamma N\nu} &= \frac{e}{4\Lambda}(f_2 - f_1) \\
C_{ZEe} &= -\frac{e}{4\Lambda}(f_2\cot\theta_W - f_1\tan\theta_W) & , \quad C_{ZN\nu} &= \frac{e}{4\Lambda}(f_2\cot\theta_W + f_1\tan\theta_W) \\
C_{WE\nu} &= C_{WNe} = \frac{e}{2\sqrt{2}\sin\theta_W\Lambda}f_2 ,
\end{align} \tag{3}$$

and the quartic interaction coupling constant, D_{VFf}, is given by

$$\begin{align}
D_{\gamma E\nu} &= -D_{\gamma Ne} = \frac{e^2\sqrt{2}}{4\sin\theta_W\Lambda}f_2 \\
D_{ZE\nu} &= -D_{ZNe} = \frac{e^2\sqrt{2}\cos\theta_W}{4\sin^2\theta_W\Lambda}f_2 .
\end{align} \tag{4}$$

The coupling of gauge bosons to excited leptons can be described by the $SU(2) \times U(1)$ invariant and CP conserving, effective Langragian,

$$\mathcal{L}_{FF} = -\bar{\Psi}^*\left[\left(g\frac{\tau^i}{2}\gamma^\mu W^i_\mu + g'\frac{Y}{2}\gamma^\mu B_\mu\right) + \left(\frac{g\kappa_2}{2\Lambda}\frac{\tau^i}{2}\sigma^{\mu\nu}\partial_\mu W^i_\nu + \frac{g'\kappa_1}{2\Lambda}\frac{Y}{2}\sigma^{\mu\nu}\partial_\mu B_\nu\right)\right]\Psi^* \tag{5}$$

In terms of the physical fields, this can be written as,

4

$$\mathcal{L}_{FF} = - \sum_{V=\gamma,Z,W} \bar{F}(A_{VFF}\gamma^{\mu}V_{\mu} + K_{VFF}\sigma^{\mu\nu}\partial_{\mu}V_{\nu})F \ . \tag{6}$$

Since we have assumed that the left- and right-handed excited leptons have the same quantum numbers under the standard gauge group, the dimension-four piece in (6) is taken vector-like. A_{VFF} is given by

$$
\begin{aligned}
A_{\gamma EE} &= -e & , & \quad A_{\gamma NN} = 0 \\
A_{ZEE} &= e\frac{(2\sin^2\theta_W - 1)}{2\sin\theta_W\cos\theta_W} & , & \quad A_{ZNN} = \frac{e}{2\sin\theta_W\cos\theta_W} \\
A_{WEN} &= \frac{e}{\sqrt{2}\sin\theta_W}
\end{aligned}
\tag{7}
$$

and K_{VFF} is given by

$$
\begin{aligned}
K_{\gamma EE} &= -\frac{e}{4\Lambda}(\kappa_2 + \kappa_1) & , & \quad K_{\gamma NN} = \frac{e}{4\Lambda}(\kappa_2 - \kappa_1) \\
K_{ZEE} &= -\frac{e}{4\Lambda}(\kappa_2\cot\theta_W - \kappa_1\tan\theta_W) & , & \quad K_{ZNN} = \frac{e}{4\Lambda}(\kappa_2\cot\theta_W + \kappa_1\tan\theta_W) \\
K_{WEN} &= \frac{e}{2\Lambda}\frac{\kappa_2}{\sqrt{2}\sin\theta_W} .
\end{aligned}
\tag{8}
$$

It is important to notice that the phenomenological model for the excited fermions described by the Lagrangians (2) and (6) has been extensively used by several experimental collaborations [8,9,10,11] to search for excited states. Therefore the results presented in this paper can be directly compared with the bounds on the excited fermion mass and compositeness scale obtained by these collaborations.

III. ANALYTICAL EXPRESSIONS

In this work we employed the on-shell-renormalization scheme, adopting the conventions of Ref. [13]. We used as inputs the fermion masses, G_F, α, and the Z-boson mass. The electroweak mixing angle is a derived quantity defined through $\sin^2\theta_W = s_W^2 \equiv 1 - M_W^2/M_Z^2$.

As a general procedure to evaluate the virtual contributions of the excited states, with couplings described by (2) and (6), we evaluated the loops in $D = 4 - 2\epsilon$ dimensions using the dimension regularization method [14] which is a gauge-invariant regularization procedure, and we adopted the unitary gauge to perform the calculations. We identified the poles at $D = 4$ ($\epsilon = 0$) and $D = 2$ ($\epsilon = 1$) with the logarithmic and quadratic dependence on the scale Λ [15]. The finite part of the loop is given by

5

$$L_{\text{finite}} = \lim_{\epsilon \to 0} \left[L(\epsilon) - R_0 \left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + 1 \right) - R_1 \left(\frac{1}{\epsilon - 1} + 1 \right) \right]$$

where $R_{0(1)}$ are the residues of the poles at $\epsilon = 0(1)$. The final result is written as

$$L = L_{\text{finite}} + R_0 \log \left(\frac{\Lambda^2}{\mu^2} \right) + R_1 \frac{\Lambda^2}{4\pi\mu^2}$$

In order to compute the loops in D dimensions in terms of the Passarino–Veltman scalar one-loop functions (see Appendix A), we used the Mathematica package FeynCalc [16]. The output of FeynCalc, in the case of the two-point functions, was checked against the results obtained by a direct analytical calculation.

Close to the Z resonance, the physics can be summarized by the effective neutral current

$$J_\mu = \left(\sqrt{2} G_\mu M_Z^2 \rho_f \right)^{1/2} \left[\left(I_3^f - 2Q^f s_W^2 \kappa_f \right) \gamma_\mu - I_3^f \gamma_\mu \gamma_5 \right] , \tag{9}$$

where Q^f (I_3^f) is the fermion electric charge (third component of weak isospin), and G_μ is the Fermi coupling constant measured via the muon lifetime. The form factors ρ_f and κ_f have universal contributions, *i.e.* independent of the fermion species, as well as non-universal parts,

$$\rho_f = 1 + \Delta\rho_{\text{univ}} + \Delta\rho_{\text{non}} , \tag{10}$$

$$\kappa_f = 1 + \Delta\kappa_{\text{univ}} + \Delta\kappa_{\text{non}} . \tag{11}$$

Excited leptons can affect the physics at the Z pole through their contributions to both universal and non-universal corrections. The universal contributions can be expressed in terms of the unrenormalized vector boson self-energies. Defining the transverse part of vacuum polarization amplitudes between the vector boson $V_1 - V_2$, $\Pi_{\mu\nu}^{V_1 V_2}(q^2)$, as

$$\Pi_{\mu\nu}^{V_1 V_2}(q^2) \equiv g_{\mu\nu} \, \Sigma^{V_1 V_2}(q^2)$$

where $V_{1,2} = \gamma$, W, and Z, we can write

$$\Delta\rho_{\text{univ}}^{\text{ex}}(s) = -\frac{\Sigma_{\text{ex}}^{ZZ}(s) - \Sigma_{\text{ex}}^{ZZ}(z)}{s - z} + \frac{\Sigma_{\text{ex}}^{ZZ}(z)}{z} - \frac{\Sigma_{\text{ex}}^{WW}(0)}{w} + 2\frac{s_W}{c_W} \frac{\Sigma_{\text{ex}}^{\gamma Z}(0)}{z} ,$$

$$\Delta\kappa_{\text{univ}}^{\text{ex}} = \frac{c_W}{s_W} \frac{\Sigma_{\text{ex}}^{\gamma Z}(z)}{z} + \frac{c_W}{s_W} \frac{\Sigma_{\text{ex}}^{\gamma Z}(0)}{z} + \frac{c_W^2}{s_W^2} \left[\frac{\Sigma_{\text{ex}}^{ZZ}(z)}{z} - \frac{\Sigma_{\text{ex}}^{WW}(w)}{w} \right] ,$$

$$\Delta r_{\text{univ}}^{\text{ex}} = \Sigma_{\text{ex}}^{\prime\gamma\gamma}(0) - \frac{c_W^2}{s_W^2} \left(\frac{\Sigma_{\text{ex}}^{ZZ}(z)}{z} - \frac{\Sigma_{\text{ex}}^{WW}(w)}{w} \right) + \frac{\Sigma_{\text{ex}}^{WW}(0) - \Sigma_{\text{ex}}^{WW}(w)}{w} - 2\frac{c_W}{s_W} \frac{\Sigma_{\text{ex}}^{\gamma Z}(0)}{z} ,$$

$$\tag{12}$$

6

where $w(z) = M_{W(Z)}^2$, $s_W(c_W) = \sin(\cos)\theta_W$ and $\Sigma' = d\Sigma/dq^2$.

The diagrams with excited lepton contributions to the self-energies are shown in Fig. 1. The final result for the transverse part of vacuum polarization $\Sigma_{Ff}^{V_1 V_2}$ contribution coming from the loop of an excited fermion with mass M and an ordinary massless fermion is

$$
\begin{aligned}
\Sigma_{Ff}^{V_1 V_2} = & \frac{1}{12\pi^2} C_{V_1 Ff} C_{V_2 Ff} \left\{ 6q^2 \Lambda^2 + q^4 \log \frac{\Lambda^2}{M^2} \right. \\
& -2q^2 M^2 - \frac{q^4}{3} + M^2(2M^2 - q^2) + (M^2 - q^2)(2M^2 + q^2) \\
& \left. \times \left[-2 + \left(1 - \frac{M^2}{q^2}\right) \log \left(1 - \frac{q^2}{M^2}\right) \right] \right\}
\end{aligned}
\tag{13}
$$

where $V_{1(2)}$ refers to the initial (final) vector boson, and the constants C_{VFf} are defined in (3) for the different vector bosons and fermions.

For the vacuum polarization, $\Sigma_{FF}^{V_1 V_2}$, coming from the loop of two excited fermions with mass M, we obtain:

$$
\begin{aligned}
\Sigma_{FF}^{V_1 V_2} = & \frac{1}{24\pi^2} \frac{q^2}{M^2} \left\{ 6 K_{V_1 FF} K_{V_2 FF} M^2 \Lambda^2 + \left[2 A_{V_1 FF} A_{V_2 FF} \right. \right. \\
& \left. + 6(A_{V_1 FF} K_{V_2 FF} + A_{V_2 FF} K_{V_1 FF})M + 3 K_{V_1 FF} K_{V_2 FF} \left(\frac{q^2}{3} + 2M^2\right) \right] M^2 \log \frac{\Lambda^2}{M^2} \\
& + 4 A_{V_1 FF} A_{V_2 FF} M^2 \left(\frac{1}{3} + \frac{2M^2}{q^2}\right) + 6(A_{V_1 FF} K_{V_2 FF} + A_{V_2 FF} K_{V_1 FF})M^3 \\
& + K_{V_1 FF} K_{V_2 FF} M^2 \left(\frac{5q^2}{3} + 4M^2\right) \\
& - 2 \frac{(4M^2 - q^2)^{1/2}}{q} \arctan \left[\frac{q}{(4M^2 - q^2)^{1/2}} \right] \left[2 A_{V_1 FF} A_{V_2 FF} M^2 \left(1 + \frac{2M^2}{q^2}\right) \right. \\
& \left. \left. + 6(A_{V_1 FF} K_{V_2 FF} + A_{V_2 FF} K_{V_1 FF})M^3 + K_{V_1 FF} K_{V_2 FF} M^2 (q^2 + 8M^2)] \right\}
\end{aligned}
$$
$$
\tag{14}
$$

For the purpose of illustration, we derived approximate expressions for the excited fermion contribution to the two-point functions, $\Sigma_{ex}^{V_1 V_2}$, in the large-M limit. For $R_Q \equiv q^2/M^2 \ll 1$, we obtain

$$
\begin{aligned}
\Sigma_{ex}^{V_1 V_2} = & \Sigma_{Ff}^{V_1 V_2} + \Sigma_{FF}^{V_1 V_2} \\
= & \frac{M^2}{12\pi^2} R_Q \left\{ 3\Lambda^2 (2 C_{V_1 Ff} C_{V_2 Ff} + K_{V_1 FF} K_{V_2 FF}) - A_{V_1 FF} A_{V_2 FF} \right. \\
& - 3(A_{V_1 FF} K_{V_2 FF} + A_{V_2 FF} K_{V_1 FF})M - 6 K_{V_1 FF} K_{V_2 FF} M^2 - 3 C_{V_1 Ff} C_{V_2 Ff} M^2 \\
& \left. + [A_{V_1 FF} A_{V_2 FF} + 3(A_{V_1 FF} K_{V_2 FF} + A_{V_2 FF} K_{V_1 FF})M + 3 K_{V_1 FF} K_{V_2 FF} M^2] \log \frac{\Lambda^2}{M^2} \right\}
\end{aligned}
$$
$$
\tag{15}
$$

We obtain in this approximation the following expressions for the universal corrections,

$$\Delta\rho(z) = \frac{\alpha}{720c_W^2 s_W^2 \pi} R_Z \, (c_W^4 + s_W^4) \left[-24 - 60\,k\,\sqrt{R_L} - 50\,f^2\,R_L - 15\,k^2\,R_L \right.$$

$$\left. + 60\,f^2\,R_L\,\log R_L + 15\,k^2\,R_L\,\log R_L \right]$$

$$\Delta\kappa = -\frac{c_W^4}{c_W^4 + s_W^4}\Delta\rho(z)$$

$$\Delta r = \frac{c_W^2}{c_W^4 + s_W^4}\Delta\rho(z)$$

(16)

where $R_Z \equiv M_Z^2/M^2$ and $R_L \equiv M^2/\Lambda^2$. For the sake of simplicity, we have assumed that $f_1 = f_2 = f$ and $k_1 = k_2 = k$.

Since we are considering non-renormalizable dimension-five operators the loops should, in principle, present poles at $D = 2$ that would generate terms that are finite when $\Lambda \to \infty$. However, we are restricting ourselves to $SU(2) \times U(1)$ gauge invariant operators, and the final results for the physical observables behave, at most, like $\log \Lambda^2/\Lambda^2$, after using the SM counterterms [see Eq. (16)]. Also, it is straightforward to verify that the new physics decouples as the new contributions in Eq. (16) vanish in the limit $R_Z \to 0$ for fixed R_L.

Corrections to the vertex $Z\bar{f}f$ give rise to non-universal contributions to ρ_f and κ_f. Excited leptons affect these couplings of the Z through the diagrams given in Fig. 2 whose results we parametrize as in Ref. [13],

$$-i\frac{e}{2s_W c_W} \left[\gamma_\mu F_{V\text{ex}}^{Zf} - \gamma_\mu \gamma_5 F_{A\text{ex}}^{Zf} - I_3^f \gamma_\mu (1 - \gamma_5)\frac{c_W}{s_W}\frac{\Sigma_{\text{ex}}^{\gamma Z}(0)}{M_Z^2} \right] \, ,$$

(17)

where the singular part proportional to $\Sigma^{\gamma Z}(0)$ has been split off, and

$$\Delta\rho_{\text{non}}^{\text{ex}} = \frac{2\,F_{A\text{ex}}^{Zf}(M_Z^2)}{I_3^f} \, ,$$

(18)

$$\Delta\kappa_{\text{non}}^{\text{ex}} = -\frac{1}{2s_W^2 Q^f} \left[F_{V\text{ex}}^{Zf} - \frac{I_3^f - 2s_W^2 Q^f}{I_3^f} \, F_{A\text{ex}}^{Zf}(M_Z^2) \right] \, .$$

(19)

There are twelve one-loop Feynman diagrams that involve the contribution of excited fermions to the three-point functions. For each diagram we define $T_i^{V_2}(q^2, M^2, M_V^2)$, $i = 1, \cdots, 12$, where V_2 is the virtual vector boson, with mass M_V, running in the loop. In our calculations, we have assumed that the ordinary fermions are massless (i.e. $m^2 \ll M^2, M_V^2$), and in this limit, $T_{7,8,9,10}^{V_2}(q^2, M^2, M_V^2) = 0$. Notice that the external fermion loops (diagrams 5–10 of Fig. 2) only contribute as half, due to the addition of the fermion wave function renormalization counterterms. We also found the relations,

8

$$T_2^{V_2}(q^2, M^2, M_V^2) = T_3^{V_2}(q^2, M^2, M_V^2) \,,$$

$$T_5^{V_2}(q^2, M^2, M_V^2) = T_6^{V_2}(q^2, M^2, M_V^2) \,,$$

$$T_{11}^{V_2}(q^2, M^2, M_V^2) = T_{12}^{V_2}(q^2, M^2, M_V^2)$$

Therefore, we can write the excited lepton contribution to the form factors $F_{V(A)\text{ex}}^{V_1 f}$, for an external vector boson V_1 as,

$$F_{V\text{ex}}^{V_1 f}(q^2) = F_{A\text{ex}}^{V_1 f}(q^2) = \frac{i s_W c_W}{e} T_{V_1 \to \bar{f} f}(q^2) \,, \tag{20}$$

with

$$
\begin{aligned}
T_{V_1 \to f^+ f^-}(q^2) = \ & T_1^\gamma(q^2, M^2, 0) + T_1^Z(q^2, M^2, M_Z^2) + T_1^W(q^2, M^2, M_W^2) \\
& + 2 \left[T_2^\gamma(q^2, M^2, 0) + T_2^Z(q^2, M^2, M_Z^2) + T_2^W(q^2, M^2, M_W^2) \right] \\
& + T_4^W(q^2, M^2, M_W^2) \\
& + \left[T_5^\gamma(q^2, M^2, 0) + T_5^Z(q^2, M^2, M_Z^2) + T_5^W(q^2, M^2, M_W^2) \right] \\
& + 2\, T_{11}^W(q^2, M^2, M_W^2) \,.
\end{aligned}
\tag{21}
$$

Our results for $T_{1,2,4,5,11}^{V_2}(q^2, M^2, M_V^2)$, in terms of the Passarino–Veltman scalar one-loop functions, are

$$
\begin{aligned}
T_1^{V_2} = \ & \frac{i}{4\pi^2 q^2} C_{V_2 F f}^2 \left\{ \left[A_{V_1 FF} \left(2\,M^6 - 3\,M^4\,M_V{}^2 + M_V{}^6 + M^2\,M_V{}^2\,q^2 + 2\,M_V{}^4\,q^2 \right) \right. \right. \\
& \left. + K_{V_1 FF} \left(2\,M\,M_V{}^4\,q^2 - 2\,M^3\,M_V{}^2\,q^2 \right) \right] \times C_0(0, 0, q^2, M^2, M_V^2, M^2) \\
& + A_{V_1 FF} \left(-2\,M^4 + M^2\,M_V{}^2 + M_V{}^4 + \tfrac{1}{3}\,M^2\,q^2 + \tfrac{2}{9}\,q^4 \right) + K_{V_1 FF} \left(M\,q^4 + 2 M M_V^2 q^2 \right) \\
& - \frac{(4\,M^2 - q^2)^{1/2}}{q} \left[A_{V_1 FF} \left(-12 M^4 + 6 M^2\,M_V^2 + 6\,M_V^4 + 10\,M^2\,Q^2 + 9\,M_V^2\,Q^2 - 4\,Q^4 \right) \right. \\
& \left. + K_{V_1 FF} \left(12\,M\,M_V^2\,Q^2 - 6\,M\,Q^4 \right) \right] \times \arctan\left[\frac{q}{(4\,M^2 - q^2)^{1/2}} \right] \\
& + \frac{q^2}{6} \left[A_{V_1 FF} \left(18\,M^2 + 9\,M_V{}^2 - 4\,q^2 \right) - 6\,K_{V_1 FF}\,M\,q^2 \right] \log\frac{\Lambda^2}{M^2} \\
& \left. - \frac{M_V^2}{(M^2 - M_V^2)} \left[A_{V_1 FF} \left(2\,M^4 - M^2\,M_V{}^2 - M_V{}^4 \right) - 2\,K_{V_1 FF}\,M\,M_V{}^2\,q^2 \right] \log\frac{M^2}{M_V^2} \right\} \\
\simeq \ & \frac{-i M^2}{144\pi^2} C_{V_2 F f}^2 \left\{ A_{V_1 FF}(126 + 117 R_V) - R_Q \left[A_{V_1 FF}(64 + 9 R_V) \right. \right. \\
& \left. + K_{V_1 FF}\,M(108 + 18 R_V) \right] + \left[-A_{V_1 FF}(108 + 54 R_V) \right. \\
& \left. + R_Q \left(24\,A_{V_1 FF} + 36\,K_{V_1 FF}\,M \right) \right] \log\frac{\Lambda^2}{M^2} \right\} \,,
\end{aligned}
\tag{22}
$$

9

where the coupling constants C_{VFf}, A_{VFF}, and K_{VFF} are given by (3), (7), and (8), respectively, and the Passarino–Veltman function $C_0(0, 0, q^2, M^2, M_V^2, M^2)$ is given in Appendix A. The approximate expression was obtained for the large-M limit, *i.e.* $R_Q = q^2/M^2 \ll 1$ and $R_V = M_V^2/M^2 \ll 1$.

$$
\begin{aligned}
T_2^{V_2} &= \frac{-i}{4\pi^2} C_{V_1 Ff} C_{V_2 Ff} \left(g_{V_2}^a + g_{V_2}^v\right) \left\{ M^2 - 2\,M_V{}^2 - 2\,q^2 - q^2 \log \frac{\Lambda^2}{M^2} + 2M_V{}^2 \log \frac{M^2}{M_V^2} \right. \\
&\quad + 2M_V{}^2 \left(M^2 - M_V{}^2 - q^2\right) C_0(0, 0, q^2, M^2, M_V^2, 0) \\
&\quad \left. + \frac{M^2 - q^2}{q^2} \left(M^2 - 2\,M_V{}^2 - q^2\right) \log\left(1 - \frac{q^2}{M^2}\right) \right\} \\
&\simeq \frac{iM^2}{8\pi^2} C_{V_1 Ff} C_{V_2 Ff} \left(g_{V_2}^a + g_{V_2}^v\right) R_Q \left(1 + 2R_V \log R_V + 2\log \frac{\Lambda^2}{M^2}\right) ,
\end{aligned}
$$

$$(23)$$

where g_V^v and g_V^a are the vector and axial coupling of the vector bosons to the usual fermions: for $V = \gamma$, $g_\gamma^v = -e$ and $g_\gamma^a = 0$; for $V = W$, $g_W^v = g_W^a = g/(2\sqrt{2})$; for $V = Z$ and $f = \nu$, $g_Z^v = g_Z^a = g/(4c_W)$; for $V = Z$ and $f = e$, $g_Z^v = g(4s_W^2 - 1)/(4c_W)$ and $g_Z^a = -g/(4c_W)$.

$$
\begin{aligned}
T_4^{V_2} &= \frac{i}{144\pi^2\,q^2} C_{V_2 Ff}^2\, g_{V_1 WW} \left\{ -36\,\Lambda^2\,q^2 + 72\,M^4 - 36\,M^2\,M_V^2 - 36\,M_V^4 - 45\,M^2\,q^2 \right. \\
&\quad + 15\,M_V^2\,q^2 + 46\,q^4 + 18\left(4\,M^6 - 6\,M^4\,M_V^2 + 2\,M_V^6 - M^4\,q^2 + 4\,M^2\,M_V^2\,q^2\right. \\
&\quad \left. + 3\,M_V^4\,q^2 - M^2\,q^4\right) \times C_0(0, 0, q^2, M_V^2, M^2, M_V^2) \\
&\quad - 6\frac{(4\,M_V^2 - q^2)^{1/2}}{q} \left(24\,M^4 - 12\,M^2\,M_V^2 - 12\,M_V^4 - 18\,M^2\,q^2 + 4\,M_V^2\,q^2\right. \\
&\quad \left. + 5\,q^4\right) \times \arctan\left[\frac{q}{(4\,M_V^2 - q^2)^{1/2}}\right] + 3\,q^2\,(18\,M^2 + 36\,M_V^2 + 5\,q^2) \log \frac{\Lambda^2}{M^2} \\
&\quad + \frac{3}{(M^2 - M_V^2)}\left(24\,M^6 - 12\,M^4\,M_V^2 - 12\,M^2\,M_V^4 - 18\,M^4\,q^2 - 36\,M_V^4\,q^2\right. \\
&\quad \left.\left. + 5\,M^2\,q^4 - 5\,M_V^2\,q^4\right) \log \frac{M^2}{M_V^2} \right\} \\
&\simeq \frac{iM^2}{288\pi^2} C_{V_2 Ff}^2\, g_{V_1 WW} \left[-72\frac{\Lambda^2}{M^2} - 18 - 36R_V + R_Q\,(103 + 144R_V + 144R_V \log R_V) \right. \\
&\quad \left. + (108 + 216R_V + 30R_Q) \log \frac{\Lambda^2}{M^2} \right] ,
\end{aligned}
$$

$$(24)$$

where $g_{V_1 WW}$ is the coupling constant of the triple vector boson vertex. For $V_1 = \gamma, Z$ is given by $g_{\gamma WW} = g s_W$ and $g_{ZWW} = g c_W$.

$$T_5^{V_2} = \frac{i}{16\left(M^2 - M_V^2\right)\pi^2} C_{V_2 F f}^2 \left(g_{V_1}^a + g_{V_1}^v\right) \left[14\, M^4 - M^2\, M_V{}^2 - 7\, M_V{}^4 \right.$$

$$-6(M^2 - M_V^2)\left(2\, M^2 + M_V{}^2\right)\log\frac{\Lambda^2}{M^2} - 6\frac{M_V^6}{M^2 - M_V^2}\log\frac{M^2}{M_V^2}\right] \tag{25}$$

$$\simeq \frac{iM^2}{16\,\pi^2} C_{V_2 F f}^2 \left(g_{V_1}^a + g_{V_1}^v\right)\left[14 + 13R_V - 6(2 + R_V)\log\frac{\Lambda^2}{M^2}\right],$$

and,

$$T_{11}^{V_2} = \frac{-i}{32\pi^2} C_{V_2 F f}\, D_{V_1 F f}\left[4\,\Lambda^2 + 15\, M^2 + 15\, M_V{}^2 - 18\left(M^2 + M_V^2\right)\log\frac{\Lambda^2}{M^2}\right.$$

$$\left. + \frac{18 M_V^4}{M^2 - M_V^2}\log\frac{M^2}{M_V^2}\right] \tag{26}$$

$$\simeq \frac{-iM^2}{32\pi^2} C_{V_2 F f}\, D_{V_1 F f}\left[4\frac{\Lambda^2}{M^2} + 15\ + 15R_V - 18(1 + R_V)\log\frac{\Lambda^2}{M^2}\right],$$

where D_{VFf} is given in (4).

In order to make a consistency check of the whole calculation, we have analyzed the effect of the excited leptons to the $\gamma \bar{f} f$ vertex at zero momentum, which is used as one of the renormalization conditions in the on-shell renormalization scheme. Taking into account the appropriate values for the constants C_{VFf} (3), A_{VFF} (7), and K_{VFF} (8), we verified that our exact result, *c.f.* Eq. 21, for the vertex $\gamma \bar{e} e$ cancels at $q^2 = 0$. This result should be expected since we are using a gauge invariant effective Lagrangian, and the QED Ward identities [17] require that the excited fermion contribution to this vertex, at zero momentum, vanishes. In the same way, we have also checked that $T_{\gamma \to \bar{\nu} \nu}(q^2 = 0) = 0$ (note that T_4 and T_{11} must change sign for external neutrinos). Moreover, we also verified that the excited fermions decouple from the vertex correction in the limit of large M.

It should be pointed out the the high energy cutoff of the loop integrals represents the maximum energy to which the effective Lagrangian is expected to apply and we have assumed that the effective operators are valid just up to the compositeness scale, Λ. Therefore, decoupling of heavy excited states occurs only when $M \to \infty$ while keeping the ratio M/Λ finite.

Finally, we present an approximated expression for the form factor $F_{V\text{ex}}^{V_1 f}$, at first order in R_Q, R_Z and R_W, which is valid for $V_1 = \gamma, Z$:

$$F_{V\,\mathrm{ex}}^{V_1 f}(q^2) = -\frac{M^2 s_W c_W}{288\pi^2 e} R_Q \left\{ 128\, A_{V_1 F'F'}\, C_{WF'f}^2 + 128\, A_{V_1 FF} \left(C_{\gamma Ff}^2 + C_{ZFf}^2 \right) \right.$$

$$+ 72\, C_{V_1 F'f'}\, C_{WF'f}\, (g_W^a + g_W^v) + 72\, C_{V_1 Ff} \left[C_{\gamma Ff}\, (g_{\gamma f}^a + g_{\gamma f}^v) + C_{ZFf}\, (g_{Zf}^a + g_{Zf}^v) \right]$$

$$+ 103\, C_{WF'f}^2\, g_{V_1 WW} + 216 \left(C_{\gamma Ff}^2\, K_{V_1 FF} + C_{ZFf}^2\, K_{V_1 FF} + C_{WF'f}^2\, K_{V_1 F'F'} \right) M$$

$$+ \left[48\, A_{V_1 F'F'}\, C_{WF'f}^2 + 48\, A_{V_1 FF} \left(C_{\gamma Ff}^2 + C_{ZFf}^2 \right) - 144\, C_{V_1 F'f'}\, C_{WF'f}\, (g_W^a + g_W^v) \right.$$

$$- 144\, C_{V_1 Ff} \left[C_{\gamma Ff}\, (g_{\gamma f}^a + g_{\gamma f}^v) + C_{ZFf}(g_{Zf}^a + g_{Zf}^v) \right] - 30\, C_{WF'f}^2\, g_{V_1 WW}$$

$$+ 72 \left(C_{\gamma Ff}^2\, K_{V_1 FF} + C_{ZFf}^2\, K_{V_1 FF} + C_{WF'f}^2\, K_{V_1 F'F'} \right) M \Bigg] \log R_L$$

$$+ R_W \left[18\, A_{V_1 F'F'}\, C_{WF'f}^2 + 144\, C_{WF'f}^2\, g_{V_1 WW} + 36\, C_{WF'f}^2\, K_{V_1 F'F'}\, M \right.$$

$$+ 144\, C_{WF'f} \left[C_{V_1 F'f'}\, (g_W^a + g_W^v) + C_{WF'f}\, g_{V_1 WW} \right] \log R_W \Bigg]$$

$$+ R_Z \left[18\, A_{V_1 FF}\, C_{ZFf}^2 + 36\, C_{ZFf}^2\, K_{V_1 FF}\, M \right.$$

$$\left. + 144\, C_{V_1 Ff}\, C_{ZFf} \left(g_{Zf}^a + g_{Zf}^v \right) \log R_Z \Bigg] \right\} .$$

$$(27)$$

From the above equation it is evident that the vertex corrections are proportional to R_Q, and therefore vanishes at $q^2 = 0$.

IV. NUMERICAL RESULTS AND DISCUSSION

The above expressions for the radiative corrections to Z physics due to excited leptons are valid for arbitrary couplings and masses. In order to gain some insight as to which corrections are the most relevant, let us begin our analyses by studying just the oblique corrections, which can also be parametrized in terms of the variables ϵ_1, ϵ_2, and ϵ_3 of Ref. [18]

$$\epsilon_1^{\mathrm{ex}} = \Delta\rho_{\mathrm{univ}}^{\mathrm{ex}}(z)$$
$$\epsilon_2^{\mathrm{ex}} = c_W^2 \Delta\rho_{\mathrm{univ}}^{\mathrm{ex}}(z) - 2s_W^2 \Delta\kappa_{\mathrm{univ}}^{\mathrm{ex}} - s_W^2 \Delta r_{\mathrm{univ}}^{\mathrm{ex}} \qquad (28)$$
$$\epsilon_3^{\mathrm{ex}} = c_W^2 \Delta\rho_{\mathrm{univ}}^{\mathrm{ex}}(z) + c_W^2 \Delta r_{\mathrm{univ}}^{\mathrm{ex}} + (c_W^2 - s_W^2)\Delta\kappa_{\mathrm{univ}}^{\mathrm{ex}}(z) .$$

Recent global analyses of the LEP, SLD, and low-energy data yield the following values for the oblique parameters [18]

$$\epsilon_1 = \epsilon_1^{\text{SM}} + \epsilon_1^{\text{new}} = (5.1 \pm 2.2) \times 10^{-3} \, ,$$

$$\epsilon_2 = \epsilon_2^{\text{SM}} + \epsilon_2^{\text{new}} = (-4.1 \pm 4.8) \times 10^{-3} \, ,\tag{29}$$

$$\epsilon_3 = \epsilon_3^{\text{SM}} + \epsilon_3^{\text{new}} = (5.1 \pm 2.0) \times 10^{-3} \, ,$$

In Fig 3, we give the attainable values for the new contributions to the ϵ parameters for different values of the excited lepton mass and couplings. As seen from this figure, requiring that the new contribution is within the limits allowed by the experimental data (29), we find that the constraints coming from oblique corrections are less restrictive than the available experimental limits. Notice that Λ being the scale of new physics, M must satisfy $M \leq \Lambda$.

As for the vertex corrections, we see in Eq. (20) that the excited leptons alter just the left-handed-lepton coupling of the Z. The new contributions to the Z widths, $\Gamma_{ee} \equiv \Gamma(Z \to e^+ e^-)$ and $\Gamma_{\text{inv}} \equiv 3\,\Gamma(Z \to \bar{\nu}\nu)$, are given by

$$\Delta\Gamma_{ee} = \alpha M_Z \frac{(s_W^2 - 1/2)}{3 s_W^2 c_W^2} \times F_V^{Ze}(z) \, ,$$

$$\Delta\Gamma_{\text{inv}} = \alpha M_Z \frac{1}{2 s_W^2 c_W^2} \times F_V^{Z\nu}(z) \, .\tag{30}$$

The theoretical values for the Z partial width generated by ZFITTER [19], for $m_{\text{top}} = 175$ GeV and $M_H = 300$ GeV, are $\Gamma_{ee} = 83.9412$ MeV and $\Gamma_{\text{inv}} = 501.482$ MeV. The most recent LEP results [22], assuming lepton universality, are $\Gamma_{\ell\ell}^{\text{LEP}}(Z \to \ell^+\ell^-) = 83.93 \pm 0.14$ MeV and for the invisible width $\Gamma_{\text{inv}}^{\text{LEP}} = 499.9 \pm 2.5$ MeV. Therefore, at 95% C.L., we should have $-0.28 < \Delta\Gamma_{ee} < 0.26$ MeV, and $-6.48 < \Delta\Gamma_{\text{inv}} < 3.32$ MeV.

We plot in Figures 4 and 5 the values of $\Delta\Gamma_{ee}$ and $\Delta\Gamma_{\text{inv}}$ attainable in this phenomenological model for some values of the compositeness scale and excited lepton masses, assuming different configurations of the weight factors $f_{1,2}$, and $\kappa_{1,2}$. Our numerical results show that the most restrictive bound on the excited fermion mass and compositeness scale comes from the comparison of $\Delta\Gamma_{ee}$ with the LEP data for this observable.

Let us compare our bounds coming from $\Delta\Gamma_{ee}$ with the ones emerging from the direct search for the excited leptons. First of all, we should point out that the direct search at LEP was just able to reach excited fermion masses up to 130 GeV [11]. On the other hand, the HERA Collaborations [9,10], looking for states that decay into a gauge boson and a usual fermion in the reaction $ep \to f^* X$, can access masses up to 250 GeV.

In Fig. 6, we present the excluded region, at 95% C.L., in the Λ versus M plane imposed by $\Delta\Gamma_{ee}$, for $f_1 = f_2 = \kappa_1 = \kappa_2 = 1$. We have further assumed that $M \leq \Lambda$, leading to the

excluded region represented by the shadowed triangle. For comparison, we also present the region excluded by the ZEUS data [10] (below and left of the dashed curve), for $f_1 = f_2 = 1$. Since we have assumed that $BR(e^* \to e\gamma) = 1$, this curve represents an upper limit for the ZEUS bound. As we can see, we were able to exclude just a small region beyond the available limit. We also show our results when we relax the condition of $M \leq \Lambda$. In the latter case, our analysis excludes all excited lepton masses with scales $\Lambda \leq 165$ GeV.

In principle, compositeness may not only generate these operators involving excited leptons which contribute to vector boson self energies and vertices at one-loop level, but it may also generate effective operators which could give tree-level contributions which we did not consider. Cancellations could then be possible between tree-level and one-loop contributions and the bounds derived here would not be applied. We assumed that it is unnatural that large cancellations occur between the tree-level and the one-loop contributions in the observables measured at M_Z scale [15].

In conclusion, we have evaluated the contribution of excited lepton states, up to the one-loop level, to the oblique variables and also to the Z width to leptons. We have compared our results with the precise data on the electroweak observables obtained by the LEP Collaborations in order to extract bounds on some of the free parameters (compositeness scale and excited lepton mass) of the phenomenological model under consideration. We also compared our results with the recent bounds obtained through the direct search for these particles. Our results show that the present precision in the electroweak parameters attained by LEP is very marginally able to constrain the parameters Λ and M beyond the present limits from direct searches.

APPENDIX A: SCALAR ONE-LOOP INTEGRALS

The relevant Passarino–Veltman functions are [20],

$$A_0(m_0^2) = -i(16\pi^2)\mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 - m_0^2} \, ,$$

$$B_0(p_1^2, m_0^2, m_1^2) = -i(16\pi^2)\mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 - m_0^2)[(k+p_1)^2 - m_1^2]} \, , \tag{A1}$$

$$C_0(p_1^2, p_{21}^2, p_2^2, m_0^2, m_1^2, m_2^2) = -i(16\pi^2)\mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 - m_0^2)[(k+p_1)^2 - m_1^2][(k+p_2)^2 - m_2^2]}$$

where $p_{21} = p_2 - p_1$

The scalar function A_0 can be written as [21],

$$A_0(m_0^2) = m_0^2 \left(\Delta - \log \frac{m_0^2}{\mu^2} + 1 \right) + 2\frac{\mu^2}{\pi(D-2)} \tag{A2}$$

where we have kept the pole at $D = 2$, and

$$\Delta = \frac{2}{4-D} - \gamma_E + \log 4\pi \tag{A3}$$

where γ_E is Euler's constant.

The B_0 and C_0 functions can be written in terms of integrals over Feynman parameters as

$$B_0(p_1^2, m_0^2, m_1^2) = \Delta - \int_0^1 dx \log \left[\frac{x^2 p_1^2 - x(p_1^2 + m_0^2 - m_1^2) + m_0^2 - i\epsilon}{\mu^2} \right] \tag{A4}$$

and

$$C_0(p_1^2, p_{21}^2, p_2^2, m_0^2, m_1^2, m_2^2) = -\int_0^1 dx \int_0^x dy \Big[p_{21}^2 x^2 + p_1^2 y^2 + (p_2^2 - p_1^2 - p_{21}^2)xy$$
$$+ (m_1^2 - m_2^2 - p_{21}^2)x + (m_0^2 - m_1^2 + p_{21}^2 - p_2^2)y + m_2^2 - i\epsilon \Big]^{-1} \tag{A5}$$

The function B_0, for some cases of interest, are

$$B_0(0, 0, M^2) = \Delta + 1 - \log \left(\frac{M^2}{\mu^2} \right) \, ,$$

$$B_0(0, M^2, M_V^2) = \Delta + 1 - \frac{(M^2 + M_V^2)}{2(M^2 - M_V^2)} \log \left(\frac{M^2}{M_V^2} \right) - \log \left(\frac{M M_V}{\mu^2} \right) \, ,$$

$$B_0(q^2, 0, M^2) = \Delta + 2 - \left(1 - \frac{M^2}{q^2} \right) \log \left(1 - \frac{q^2}{M^2} \right) - \log \left(\frac{M^2}{\mu^2} \right) \, ,$$

$$B_0(q^2, M^2, M^2) = \Delta + 2 - 2\frac{(4M^2 - q^2)^{1/2}}{q} \arctan \left[\frac{q}{(4M^2 - q^2)^{1/2}} \right] - \log \frac{M^2}{\mu^2}$$

The functions C_0, for some cases of interest, are

$$C_0(0,0,0,M^2,0,M^2) = -\frac{1}{M^2} \; ,$$

$$C_0(0,0,0,M^2,M_V^2,0) = -\frac{1}{(M^2-M_V^2)} \log\left(\frac{M^2}{M_V^2}\right) \; ,$$

$$C_0(0,0,0,M^2,M_V^2,M^2) = -\frac{1}{(M^2-M_V^2)^2}\left\{M^2 - M_V^2\left[1+\log\left(\frac{M^2}{M_V^2}\right)\right]\right\} \; ,$$

$$\begin{aligned}
C_0(0,0,q^2,M^2,M_V^2,0) = &\; \frac{1}{q^2}\left[\log\left(\frac{M^2-q^2}{M_V^2}\right)\log\left(\frac{M^2-q^2}{M_V^2}-1\right)\right. \\
&- \log\left(\frac{M^2}{M_V^2}-1\right)\log\left(\frac{M^2}{M_V^2}\right) \\
&\left.+i\pi\log\left(1-\frac{q^2}{M^2}\right) - \mathrm{Li}_2\left(\frac{M^2}{M_V^2}\right) + \mathrm{Li}_2\left(\frac{M^2-q^2}{M_V^2}\right)\right] \; ,
\end{aligned}$$

$$\begin{aligned}
C_0(0,0,q^2,M^2,M_V^2,M^2) = &\; \frac{1}{q^2}\left\{-2\pi\arctan\left[\frac{q(4M^2-q^2)^{1/2}}{2(M^2-M_V^2)-q^2}\right]\right. \\
&+4\arctan\left[\frac{(4M^2-q^2)^{1/2}}{q}\right]\arctan\left[\frac{q(4M^2-q^2)^{1/2}}{2(M^2-M_V^2)-q^2}\right] \\
&-\log\left(\frac{M^2}{M_V^2}\right)\log\left[\frac{(M^2-M_V^2)^2}{(M^2-M_V^2)^2+M_V^2 q^2}\right] \\
&-\mathrm{Li}_2\left(\frac{M^2 q^2}{(M^2-M_V^2)^2+M_V^2 q^2}\right) + \mathrm{Li}_2\left(\frac{M_V^2 q^2}{(M^2-M_V^2)^2+M_V^2 q^2}\right) \\
&+\mathrm{Li}_2\left(\frac{\xi^*}{2(M^2-M_V^2)-\xi}\right) - \mathrm{Li}_2\left(\frac{-\xi^*}{2(M^2-M_V^2)-\xi^*}\right) \\
&\left.+\mathrm{Li}_2\left(\frac{\xi}{2(M^2-M_V^2)-\xi^*}\right) - \mathrm{Li}_2\left(\frac{-\xi}{2(M^2-M_V^2)-\xi}\right)\right\} \; ,
\end{aligned}$$

$$\begin{aligned}
C_0(0,0,q^2,M^2,0,M^2) = &\; \frac{1}{q^2}\left\{-2\pi\arctan\left[\frac{q(4M^2-q^2)^{1/2}}{2M^2-q^2}\right] - \mathrm{Li}_2\left(\frac{M^2 q^2}{M^4}\right)\right. \\
&+4\arctan\left[\frac{(4M^2-q^2)^{1/2}}{q}\right]\arctan\left[\frac{q(4M^2-q^2)^{1/2}}{2M^2-q^2}\right] \\
&+\mathrm{Li}_2\left(\frac{\xi^*}{2M^2-\xi}\right) - \mathrm{Li}_2\left(\frac{-\xi^*}{2M^2-\xi^*}\right) + \mathrm{Li}_2\left(\frac{\xi}{2M^2-\xi^*}\right) \\
&\left.-\mathrm{Li}_2\left(\frac{-\xi}{2M^2-\xi}\right)\right\} \; ,
\end{aligned}$$

$$\tag{A7}$$

where $\xi = q^2 + iq(4M^2-q^2)^{1/2}$, and $\mathrm{Li}_2(x)$ is the dilogarithm or Spence's function, defined as

$$\mathrm{Li}_2(x) = -\int_0^1 \frac{dt}{t}\log(1-xt) \; ,$$

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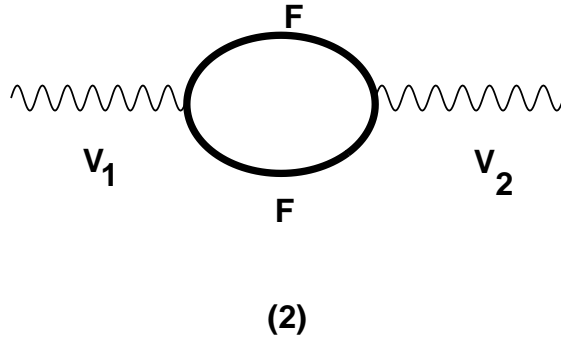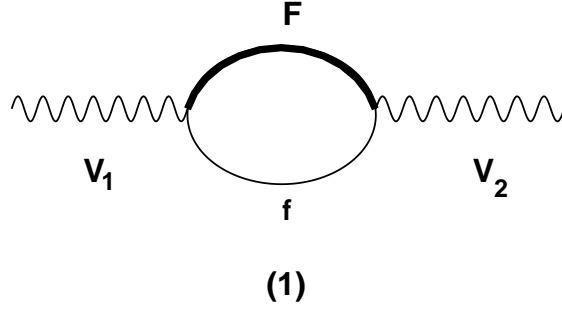
FIGURES

F

V₁

f

V₂

(1)

F

V₁

F

V₂

(2)

FIG. 1. Feynman diagrams leading to contribution of the excited leptons to the two-point functions

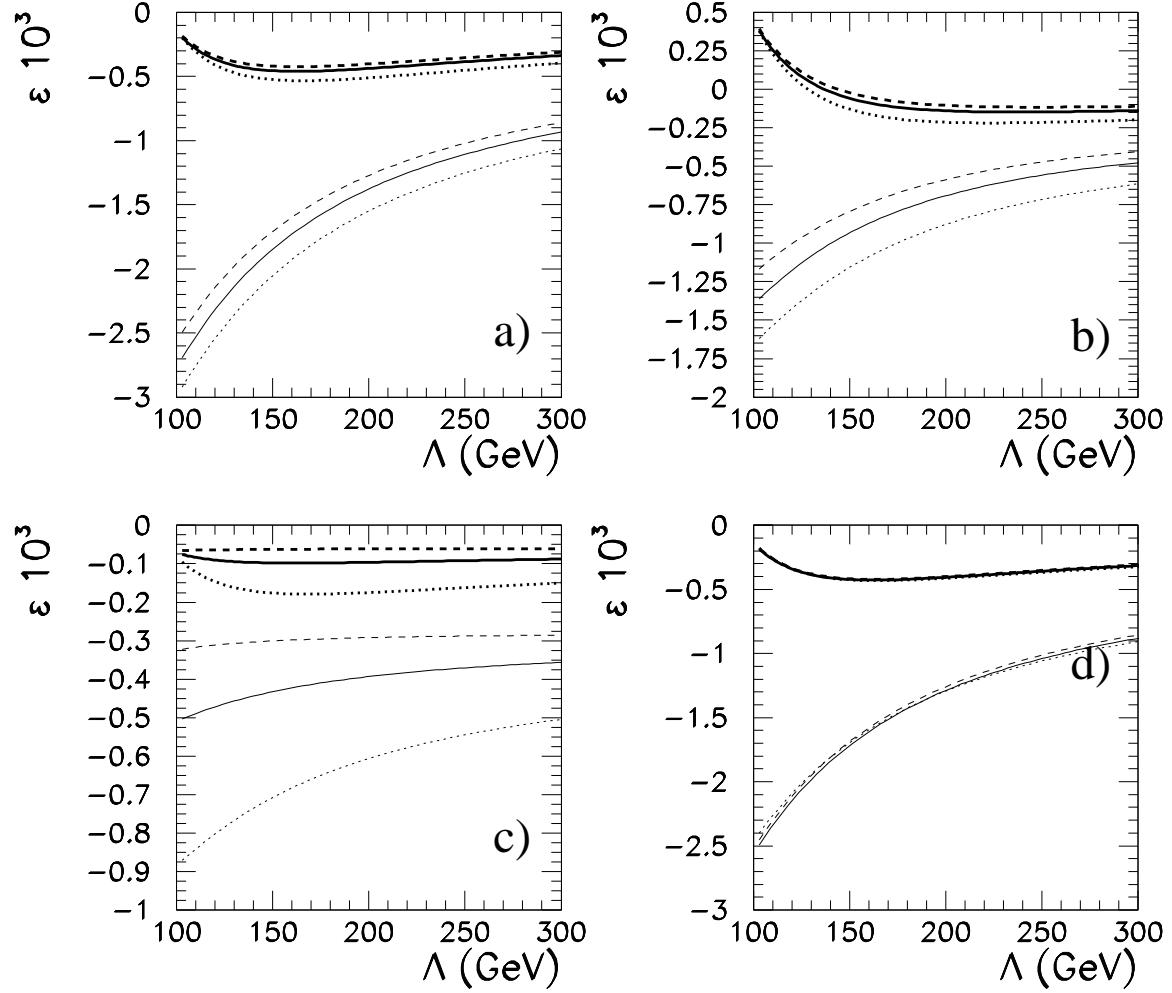FIG. 2. The contribution of the excited leptons to the three-point functions

21

FIG. 3. Attainable values for the new contributions to the ϵ's parameters in the model as a function of the scale Λ. The solid lines correspond to ϵ_1, the dashed ones to ϵ_2 and the dotted ones to ϵ_3. The thin (thick) lines correspond to excited lepton mass value of $M = 100$ (200) GeV. We have assumed different configurations of the weight factors $(f_1, f_2, \kappa_1, \kappa_2)$: (a) $= (1, 1, 1, 1)$; (b) $= (1, -1, 1, -1)$; (c) $= (1, 0, 1, 0)$; (d) $= (0, 1, 0, 1)$
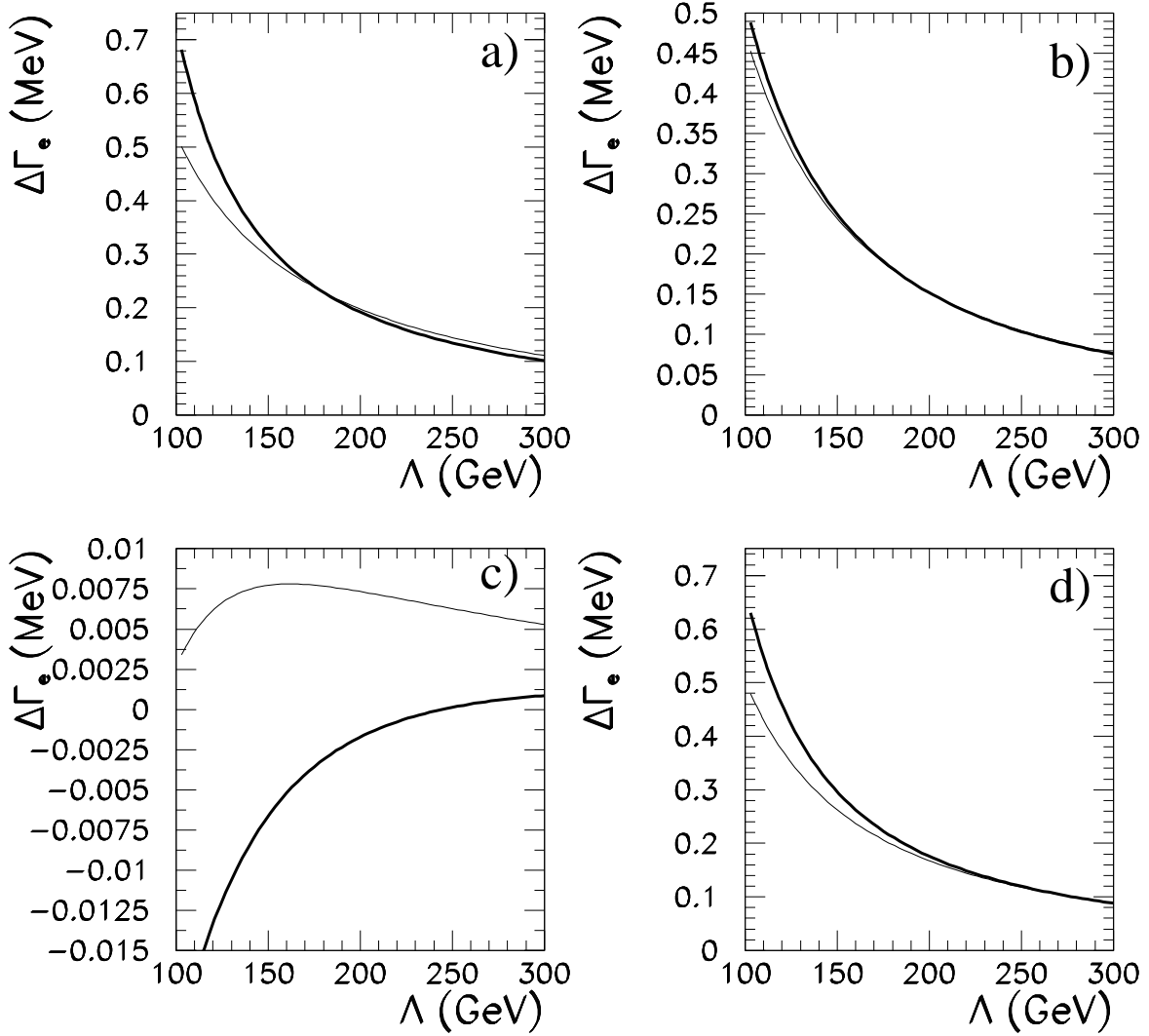
FIG. 4. Attainable values for the new contributions to the width $\Gamma(Z \to \ell^+\ell^-)$ in the model as a function of the scale Λ. The thin (thick) line correspond to excited lepton mass value of $M = 100$ (200) GeV, for configurations of the weight factors as in Fig. 3.
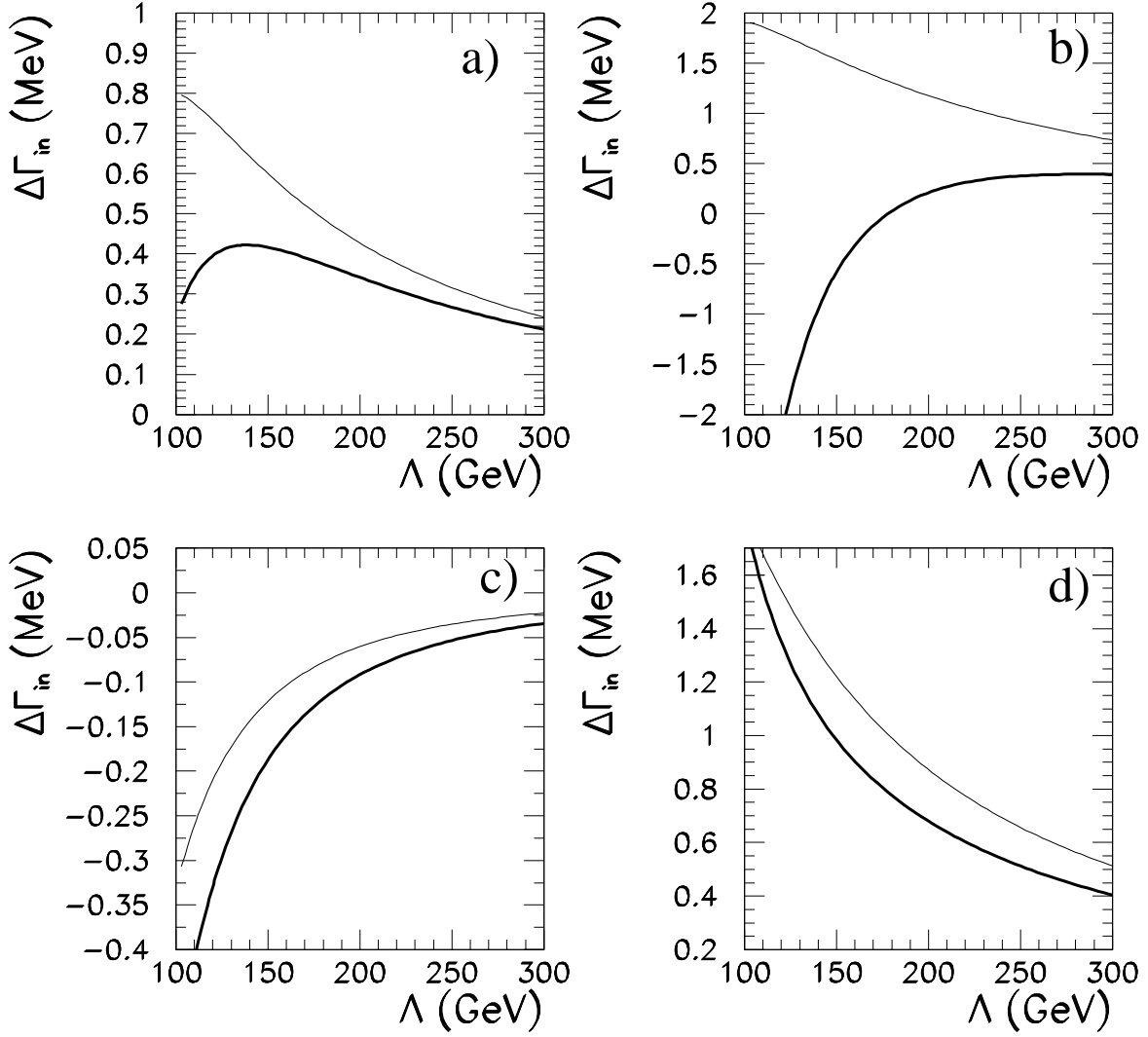
FIG. 5. Attainable values for the new contributions to the invisible Z width in the model as a function of the scale Λ. The thin (thick) line correspond to excited lepton mass value of $M = 100$ (200) GeV, for configurations of the weight factors as in Fig. 3.
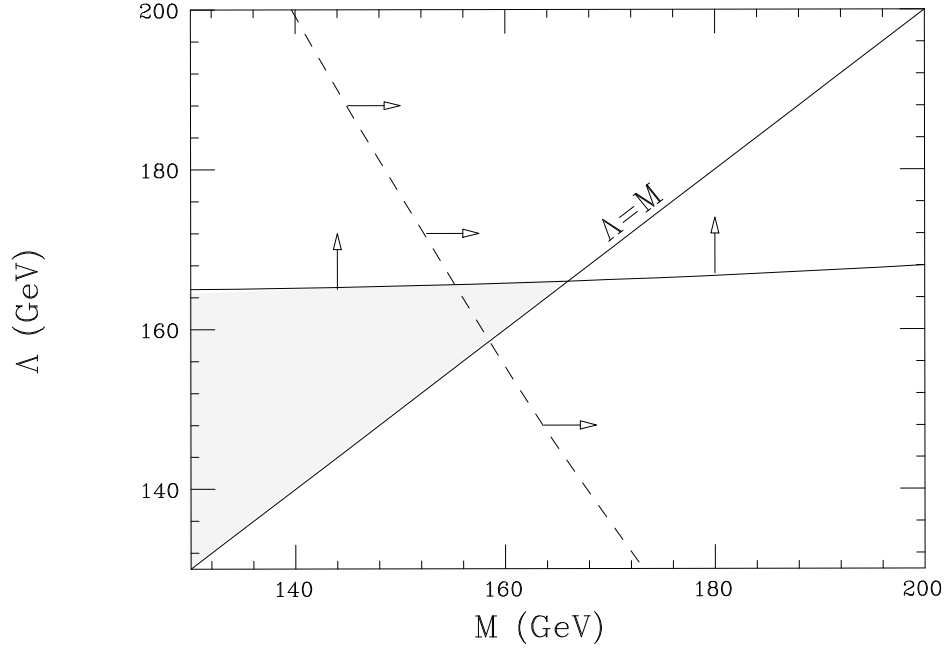
FIG. 6. Excluded regions in the Λ versus M plane from the bounds on $\Delta\Gamma_{ee}$ (shadowed area), and from ZEUS data [10] (below and left of the dashed curve), at 95% C.L.